

Indian Statistical Institute, Bangalore

M. Math (Hons.) Second Year

First Semester - Fourier Analysis

Midterm Exam

Date: 05th September 2026

Maximum marks: 30

Duration: 2 hours

Answer any 6 each question carries 5 marks

1. Let (ϕ_n) be a nonnegative summability kernel and $f \in L^p$ for $1 \leq p < \infty$. Prove that $\phi_n * f \rightarrow f$ in L^p .
2. If f is differentiable with $\|f'\|_2 \leq 1$, show that $\|f - s_n\|_\infty \leq \frac{2}{\sqrt{n}}$ for all $n \geq 1$.
3. Prove Riemann-Lebesgue Lemma for $f \in L^1(\mathbb{R})$.
4. Let ϕ be a step function. Prove that $(n[\hat{\phi}(n) + \hat{\phi}(-n)])$ is bounded.
5. Prove Fejer's Theorem for Cesaro average.
6. For $f \in L^1(\mathbb{T})$ and $g \in L^\infty(\mathbb{T})$, find $\lim_{n \rightarrow \infty} \int f(t)g(nt)$.
7. (a) Prove that Poisson kernel $P_r(\theta) = \frac{1-r^2}{1-2r \cos \theta + r^2}$ (Marks 2) .
(b) Suppose $f \in L^1(\mathbb{T})$ is bounded and monotonic in an open interval I of $\mathbb{T}$. Prove that $s_n(x)$ converges for all $x \in I$.
8. Find a non-zero solution $u(r, \theta)$ on the open unit disc such that $\Delta u = 0$ but $u(r, \theta) \rightarrow 0$ as $r \rightarrow 1$ for every θ .